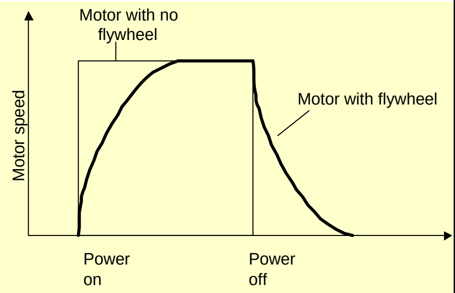
Inductors and Capacitors – Energy Storage Devices

Aims:
To know:
- Basics of energy storage devices.
- Storage leads to time delays.
- Basic equations for inductors and capacitors.

To be able to do describe:
- Energy storage in circuits with a capacitor.
- Energy storage in circuits with an inductor.

Energy Storage and Time Delays

- Changes in resistor networks happen "instantaneously"
- No energy is stored in a resistor network (only dissipated)
- Devices which store energy introduce time delays
  - Time to store energy
  - Time to release energy
  - Example – Flywheel storage

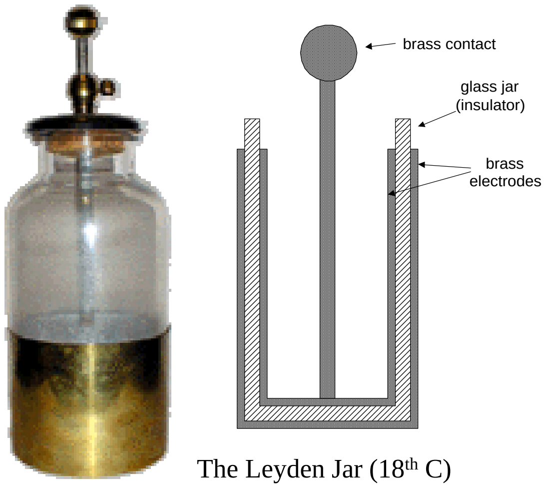


- Electronic components that store energy will force us to think about how currents and voltages change with time

Capacitor

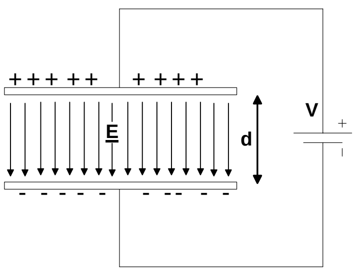
- A device to store charge.
- Excess charges generate electrostatic fields.
- Electrostatic fields are associated with energy

- Capacitors are devices to generate a well-defined electrostatic field



The Leyden Jar (18th C)

Simplest geometry



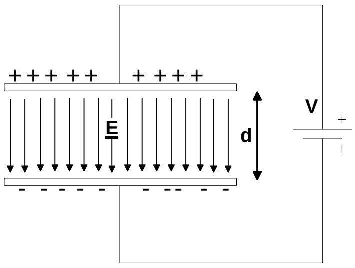
Parallel plate capacitor (schematic)

You will learn that if you take any closed surface surrounding an isolated charge, the electric field multiplied by the area of the surface is proportional to the value of the charge:

$$Q = \varepsilon_0 EA \qquad \textit{(This is Gauss's Theorem)}$$

So for this geometry (where we assume that E is constant over the entire area of the plates)

Simplest geometry



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So for this geometry (where we assume that E is constant over the entire area of the plates)

$E = V/d$, so

$$Q = \frac{\varepsilon_0 VA}{d}$$

or $Q = CV$, where

$$\mathrm{C} = \frac{\varepsilon_0 A}{d} \text{ coulomb per volt}$$

Johann Carl Friedrich Gauss
(1777 - 1855)

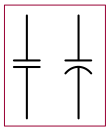
One of the 19th century's great minds

Primarily a mathematician, he worked in Göttingen on many fundamental aspects of mathematical physics and statistics

Capacitance

$Q = CV$, where

$$C = \frac{\varepsilon_0 A}{d} \text{ coulomb per volt}$$



Circuit symbols

C is called the CAPACITANCE of the device.
This is a property of the configuration of the electrodes
The unit C V^{-1} is called the **FARAD** (F).

1 Farad is a very large capacitance and capacitors commonly used range from a few pF through nF and μF to ~1 mF

example: A=10 cm x 10 cm and d = 1mm

A capacitor stores a well defined amount of charge proportional to the voltage. When it is disconnected from the battery it will store the charge indefinitely.

This is NOT like a battery where the amount of charge GENERATED is independent of voltage. You can only take out of a capacitor what you put into it

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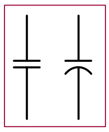

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$$C = \frac{\varepsilon_0 A}{d} \text{ F}$$

$$C = \frac{8.85\times10^{-12}\times10^{-1}\times10^{-1}}{10^{-3}} = 8.85\times10^{-11} \text{ F}$$

$$C = 88.5 \text{ pF}$$

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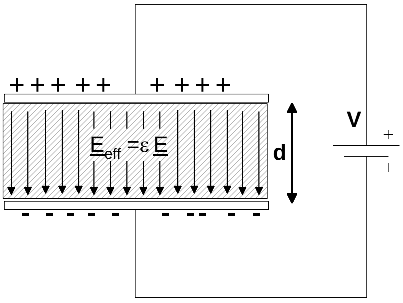
Michael Faraday
(1791 - 1867)

A great experimentalist and populariser of science.

He is most famous for his work on magnetic induction, but also did fundamental work related to electrolysis

He worked at the Royal Institution (one of the first scientific research institutes) and established the Christmas Lectures on science for young people which are still running.

Dielectrics



If we place an insulating material between the plates of our capacitor, the effective field increases*.

$$\underline{E}_{eff} = \varepsilon \underline{E}$$

Where ε is a dimensionless property of the material called the **dielectric constant** or **relative permittivity.**
ε is *usually* > 1
e.g. for glass ε = 8

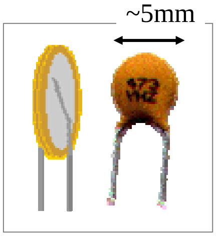
This increases the capacitance: $C = \frac{\varepsilon \varepsilon_0 A}{d}$ F

* This is because the electron cloud round each atom in the material is distorted by the applied field and this generates an additional field (this is called the displacement field, D)

Practical Capacitors

Practical capacitors try to squeeze as much capacitance as possible into the smallest physical volume:

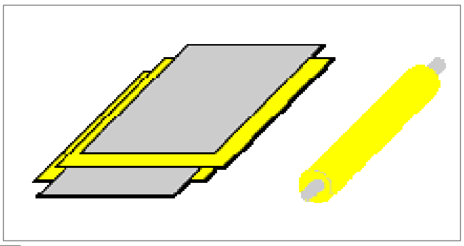
- Large area
- Small separation
- High dielectric constant insulator

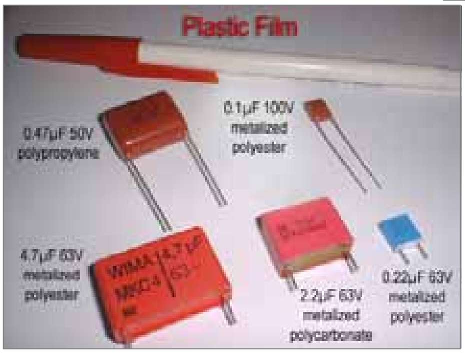


e.g. Ceramic disc capacitor
Electrodes are metal (Al, Ag) evaporated onto two sides of disk of very high permittivity ceramic

Practical Capacitors

Plastic film capacitor
Electrodes are metal (Al, Ag) evaporated onto both sides of a long ribbon of very thin Mylar foil which is stacked in a block or rolled up like a Swiss Roll into a small cylinder





Many other types for a wide range of applications

Energy in Capacitors

Imagine a capacitor C charged to a voltage V

If you push into the capacitor a small amount of charge, dQ, then the energy increases by an amount d$W = V$dQ *(energy = charge x voltage)*

At the same time the voltage increases by an amount dV, where d$Q = C$dV
So d$W = CV$dV.

To get the total energy stored in a capacitor we need to integrate this expression:

$$W = \int_0^V CVdV$$

Compare this with a battery, where W = QV

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$$W = \frac{1}{2}CV^2 \quad \text{or} \quad \mathrm{W} = \frac{1}{2}\mathrm{QV}$$

Compare this with a battery, where W = QV

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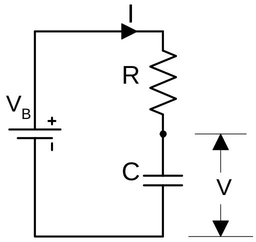
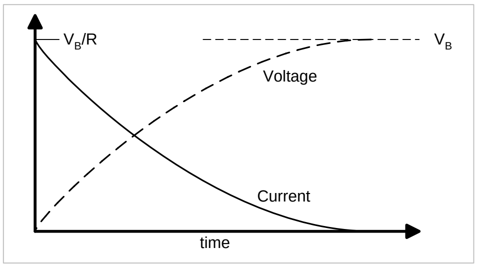
Charging a capacitor



When you first connect a battery to a capacitor:
- The voltage across the capacitor is ZERO
- The current is high (V_B/R)

When the capacitor is fully charged:
- The voltage across the capacitor is V_B
- The current is ZERO

For capacitors:
Current leads Voltage



$$Q = CV$$

$$i = \frac{dQ}{dt}$$

so $i = C\frac{dV}{dt}$

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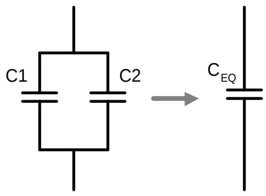
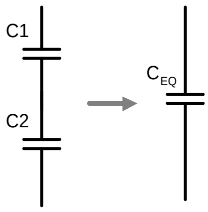
Series and parallel capacitors



For parallel capacitors, V is the same, so total charge is given by



For series capacitors, the CHARGE on each capacitor must be the same and equal to the net charge.
[The centre electrode has a net charge of zero]

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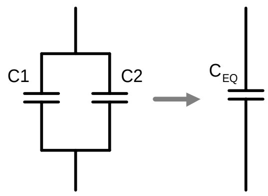
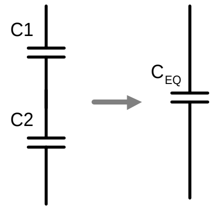
Series and parallel capacitors



For parallel capacitors, V is the same, so total charge is given by

$$Q_{TOT} = C_{EQ}V = Q_1 + Q_2 = C_1V + C_2V$$

Hence: $\underline{C_{EQ} = C_1 + C_2}$



For series capacitors, the CHARGE on each capacitor must be the same and equal to the net charge.
[The centre electrode has a net charge of zero]

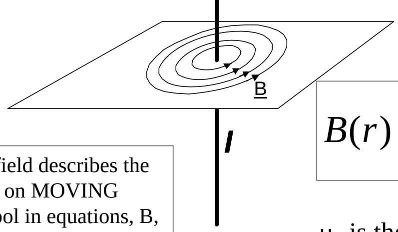
$$V_{TOT} = \frac{Q_{TOT}}{C_{EQ}} = V_1 + V_2 = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$$Q_{TOT} = Q_1 = Q_2$$

Hence: $\underline{\frac{1}{C_{EQ}} = \frac{1}{C_1} + \frac{1}{C_2}}$

Inductors: Energy Storage in Magnetic Fields

Flowing electric currents create magnetic fields



Right hand rule: current with thumb, field with fingers

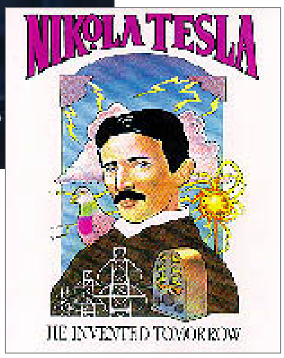
$$B(r) = \frac{\mu_0 I}{2\pi r}$$

The magnetic field describes the magnetic force on MOVING charges. Symbol in equations, B, units TESLA, (T).

μ_0 is the **permeability of free space** a fundamental constant that relates magnetism to force and energy.

$\mu_0 = 4\pi \times 10^{-7}$ T A^{-1} m

Check that $\frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ gives the value of the velocity of light

Nikola Tesla

(1856 - 1943)

Serbian immigrant to the USA. Considered to be more of an inventor than a scientist and is credited with the idea of using AC for power transmission.

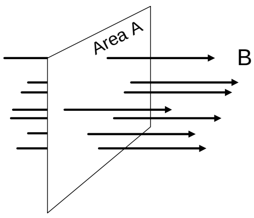
Much given to spectacular demonstrations of high voltage sparks, he became one of the first scientific superstars in the US.

Magnetic Flux and Inductance

The total amount of magnetic field crossing a surface is called the flux:

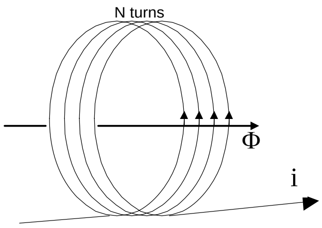
If the field is uniform, the flux is given by

$$\Phi = BA \text{ T m}^2$$



For any general coil of N turns carrying current i the total amount of flux generated is defined as

$$\Phi = \frac{Li}{N}$$



Where L is a parameter depending only on the shape and number of turns of the coil called INDUCTANCE.
Units: T m^2 A^{-1} or HENRY (symbol H)

Joseph Henry
(1797 - 1878)

Born in upstate New York he worked on electromagnetism and inductance in Albany and Princeton.

Was appointed the first Secretary of the Smithsonian Institution in Washington in 1864

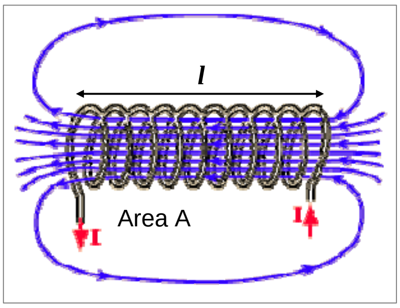
Solenoids

The magnetic field can be concentrated by forming the wire into a coil or solenoid. For a long solenoid:

$$B = \frac{\mu_0 N i}{l} \quad \text{so} \quad \Phi = \frac{\mu_0 N A i}{l}$$

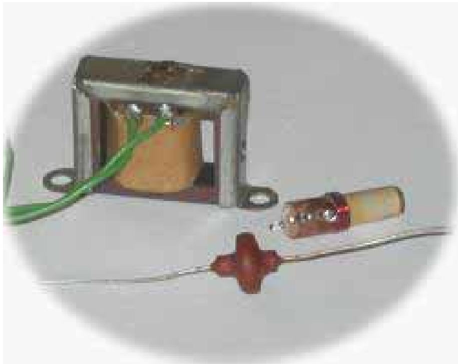
and

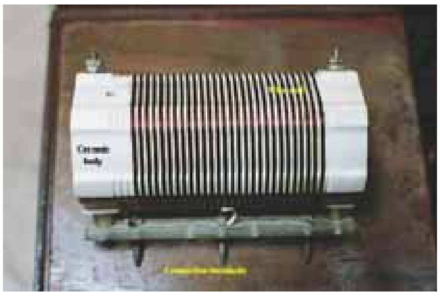
$$L = \frac{\Phi N}{i} = \frac{\mu_0 N^2 A}{l} \quad \text{Henry}$$

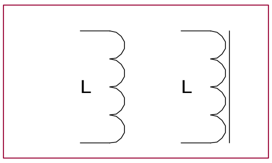


Adding a ferromagnetic (e.g. iron) CORE into the coil can increase the flux for a given current and so increase the inductance

Practical inductors



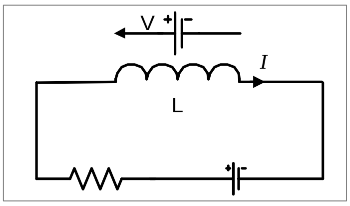




Circuit symbols

Back e.m.f.

When we try to change the current passing through an inductor the increasing magnetic field **induces** a reverse voltage which tries to oppose the change.



This depends on the inductance and how fast the current is changing:

$$V = -L\frac{dI}{dt}$$

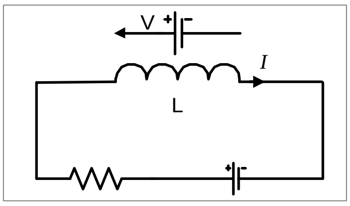
- This is Lenz's Law which is based on Faraday's laws of magnetic induction.

So we have to do work to overcome this back e.m.f. and pass current through an inductor

– we are storing energy in the magnetic field.

Energy in inductors

$$V = -L\frac{dI}{dt}$$



So in a short time *dt* we have to do a small amount of work

$$dW = IVdt = LIdI$$

to overcome the back e.m.f.

Thus the total energy required to increase the current from 0 to *I* is

$$W =$$

This is the energy stored in an inductor

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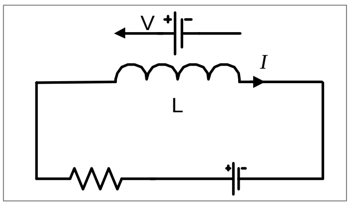
Energy in inductors

$$V = -L\frac{dI}{dt}$$



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to overcome the back e.m.f.

Thus the total energy required to increase the current from 0 to *I* is

$$W = \int_0^I LIdI = \frac{1}{2}LI^2$$

This is the energy stored in an inductor

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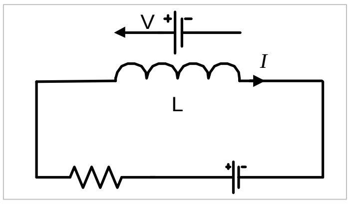
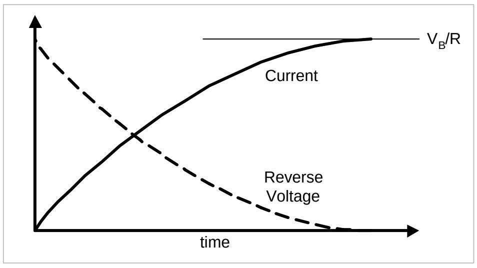
“Charging” an Inductor

When you first connect a battery to an inductor:
•The current through the inductor is ZERO
•The back e.m.f. is high

When the inductor is carrying the full current:
•The voltage across the inductor is ZERO
•The current is high

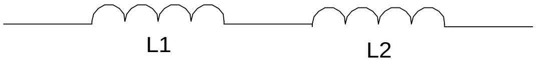


For inductors:
Voltage leads Current

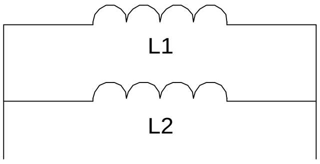


Inductors in Series and Parallel

For inductors in series

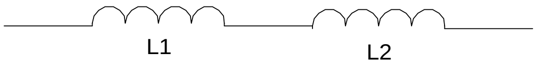


For inductors in parallel



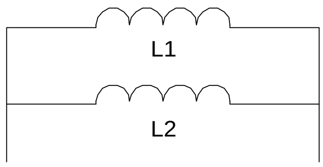
Inductors in Series and Parallel

For inductors in series



$$L_{EQ} = L_1 + L_2$$

For inductors in parallel



$$\frac{1}{L_{EQ}} = \frac{1}{L_1} + \frac{1}{L_2}$$

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Summary of Capacitor and Inductor Formulae

	Capacitor	**Inductor**	**Resistor**
I-V relationship	$I = C\frac{dV}{dt}$	$V = -L\frac{dI}{dt}$	$V = IR$
Stored energy	$W = \frac{1}{2}CV^2$	$W = \frac{1}{2}LI^2$	0
Dissipated energy	0	0	$P = IV$
Series equivalent	$1/\left(\frac{1}{C1} + \frac{1}{C2}\right)$	$L1 + L2$	$R1 + R2$
Parallel equivalent	$C1 + C2$	$1/\left(\frac{1}{L1} + \frac{1}{L2}\right)$	$1/\left(\frac{1}{R1} + \frac{1}{R2}\right)$
Current/voltage timing	Current leads voltage	Voltage leads current	Current in phase with voltage

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